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PROBABILISTIC ANALYSIS OF STRUCTURE MODELS USING TARGET RANDOM SAMPLING (TRS)

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ABSTRACT: The work presents testing methods of sensitivity and reliability of mechanical or structural systems. All computations concerned the case of Zigler system, a simple model of a compressed column involving two random variables only. A conclusion was drawn that the standard Monte Carlo (MC) method, its reduction variants, the response surface method (RSM) and target random sampling (TRS) allow to assess the sensitivity of structural response to the variation of random structural parameters. Sensitivity assessment was proposed on the basis of Sobol indices and the analysis of limit state surface intersections.

Keywords: reliability, sensitivity, response surface method, Sobol indices, Ziegler column

1. INTRODUCTION

In recent times the probabilistic methods become more and more decisive in structural analysis and design. While the methods are majorly aimed at structural reliability estimation, attempts are made of their enhancement to the tasks of optimization and sensitivity analysis. The latter issue is the one the work is focused on.

The analysis of both structural reliability and sensitivity is conducted by means of a series of methods, of a classic status, e.g. Monte Carlo (MC) simulation incorporating variance-reduction techniques. Other widespread methods in these fields are: polynomial chaos expansion, Linear Regression Analysis (LRA), Analysis of Variance (ANOVA), Response Surface Method (RSM) and many others.

The domain of sensitivity analysis methods splits into local and global approaches (Ref. 1). Sensitivity assessment due to a single parameter change is possible with the help of: Differential Analysis (DA), the One-At-a-Time technique (OAT), Importance Factors (IF) or Sensitivity Index (SI) (Ref. 2). While a nonlinear model is valid or high differences occur between the modification results of different parameters local methods should be replaced by the global ones (Ref. 3).

There does not exist a single ultimate sensitivity measure. Various methods incorporate a linear or linearized limit state function to capture linear impact of basic variables. Here sensitivity factors are equal to partial derivatives of the limit state function with respect to basic variables at either the mean value point or the design point (Ref. 4). There was Sobol who proposed one of the most advanced sensitivity indices (Ref. 5). Probabilistic sensitivity of structural limit states with the use of histograms is displayed e.g. in Ref. 6.

The stochastic sensitivity methods allow to analyse a broad domain of structural types. Attention is paid to imperfection-prone structures, e.g. truss and framed structures analysed in Refs 7-14 or thin-walled members (Refs 15-19). A vast literature on plates and shells was not considered here. Other fields are worth noting too, e.g. fracture mechanics (Ref. 20) or sensitivity analysis of drilling platforms (Ref. 21).

The work involves Sobol (Ref. 5) sensitivity index determination, i.e. both cases: the first order and the total sensitivity index. The sensitivity indices were determined on the basis of curves - intersection lines of structural response surfaces and the planes of constant design variable values. The work employs a classic Ziegler column case to test the chosen approaches.

2. NUMERICAL EXAMPLES – ZIEGLER COLUMN

Ziegler columns (Fig. 1) were analysed in a number of papers, e.g. Refs 22, 23. It concerns an axially loaded (λP) column clamped at its bottom. Two hinges are introduced in the system: the first at the base, the second at the mid-height, both spring-supported by constant stiffness springs k_1 and k_2, respectively (Fig. 1). The spring stiffnesses are assumed Gaussian random variables $k_1 = \bar{k}_1(1+\alpha_1)$, $k_2 = \bar{k}_2(1+\alpha_2)$, of the following mean values $m_{\alpha_1} = 0.0$ and standard deviations $\sigma_{\alpha_1} = 0.2$.

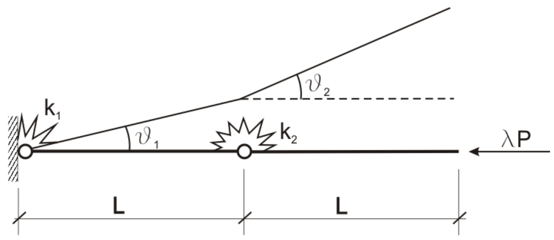

Fig.1 Ziegler column (Ref. 24)

The explicit limit state function takes the form

$$g(\alpha_1,\alpha_2) = \frac{1}{2}\bar{k}_1(1+\alpha_1) + \bar{k}_2(1+\alpha_2) - \\ -\frac{1}{2}\sqrt{\bar{k}_1^2(1+\alpha_1)^2 + 4\bar{k}_2^2(1+\alpha_2)^2} - \lambda \tag{1}$$

Assuming that $k_1 = k_2 = \bar{k}_1 = \bar{k}_2 = 1$ [kNm / rad], $L = 1$ m and $P = 1$ kN the limit load multiplier was equal to $\lambda = 0{,}75\lambda_E = 0{,}375(3-\sqrt{5})$. It was finally assumed that the variables α_1 and α_2 are uncorrelated (while Gaussian, they were independent).

3. ANALYSIS OF THE RESULTS SCATTER

The first method allowing for a simple and straightforward analysis of the model mechanical response is the so-called scatter image technique. Fig. 2 presents the relations between the system response variable g and stiffnesses: k_1 and k_2.

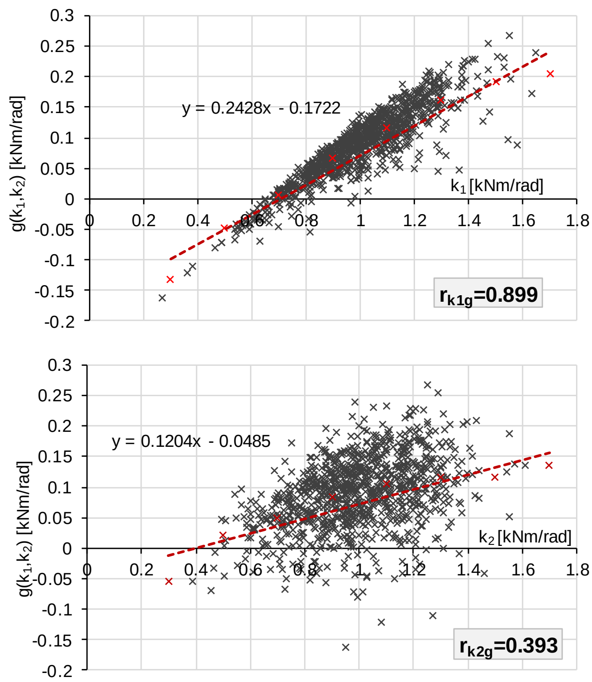

Fig. 2 The shape of result scatter cloud due to variables k_1 and k_2.

In the case of each cloud of points the estimator of linear correlation coefficient was determined by means of the relation:

$$r_{k_m g} = \frac{\sum_{i=1}^{N}\left(k_{mi} - \bar{k}_m\right)\left(g_i - \bar{g}_i\right)}{\sqrt{\sum_{i=1}^{N}\left(k_{mi} - \bar{k}_m\right)^2}\sqrt{\sum_{i=1}^{N}\left(g_i - \bar{g}_i\right)^2}}, \qquad m = 1,2 \tag{2}$$

In the case of scatter clouds of the variables k_1 and k_2. the coefficient values were equal $r_{k1g} = 0.899$ and $r_{k2g} = 0.393$ respectively.

In Figure 2 the red marks refer to the mean values of the response variable based on samples related to selected ranges of the analysed stiffness. The variability domain was divided into eight equal sub-domains. The approximation straight line was drawn according to the obtained points, the line slope is an attempt to visually assess the model sensitivity. The increase of the slope marks the sensitivity rise to the variation of the related parameter.

The clouds and correlation coefficients make it possible to conclude that stiffness k_1 is the variable the system is highly sensitive to. The shape of the point cloud in the first image is relatively easy to interpret while the cloud of points in the second figure resembles a uniform distribution.

4. THE SOBOL SENSITIVITY INDICES

The Sobol sensitivity indices of the first order (S_i) and of total sensitivity (S_{T_i}) were determined in the case of stiffnesses k_1 and k_2. The first order sensitivity index S_i expresses a direct impact of a given parameter to the model response, the total sensitivity index S_{T_i} accounts for the impact of a parameter and its interaction with the others to the output response. Both first order and total sensitivity indices are determined by the formulae:

$$S_i = \frac{V\left[E\left(Y \mid X_i\right)\right]}{V(Y)} = \frac{y_A \cdot y_{C_i} - f_0^2}{y_A \cdot y_A - f_0^2} = \frac{(1/N)\sum_{j=1}^{N} y_A^{(j)} y_{C_i}^{(j)} - f_0^2}{(1/N)\sum_{j=1}^{N}\left(y_A^{(j)}\right)^2 - f_0^2} \tag{3}$$

$$S_{T_i} = \frac{V\left[E\left(Y \mid X_{\sim i}\right)\right]}{V(Y)} = 1 - \frac{y_B \cdot y_{C_i} - f_0^2}{y_A \cdot y_A - f_0^2} = 1 - \frac{(1/N)\sum_{j=1}^{N} y_B^{(j)} y_{C_i}^{(j)} - f_0^2}{(1/N)\sum_{j=1}^{N}\left(y_A^{(j)}\right)^2 - f_0^2} \tag{4}$$

where

$$f_0^2 = \left(\frac{1}{N}\sum_{j=1}^{N} y_A^{(j)}\right)^2 .$$

The detailed description of formulae (3) and (4) is included in [Oziębło, 2018].

The considered sensitivity indices of variables k_1 and k_2 based on formulae (3) and (4) were equal to $S_{k_1} = 0.7974$, $S_{T_{k_1}} = 0.8369$, and $S_{k_2} = 0.1898$, $S_{T_{k_2}} = 0.2292$, respectively. The results mark a significantly higher impact of the supporting spring stiffness on the system mechanical response.

5. MONTE CARLO METHODS

In order to assess the failure probability by means of direct Monte Carlo method a population of 10^6 samples was generated. This led to the estimation of $p_f = 0.0737$, the reliability index estimated as $\beta = 1.449$. This results serves as a reference level.

These results were compared to the results of Monte Carlo stratified sampling (MC-SS). The variable space was split into 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2, 15^2, 20^2, 25^2 and 30^2 equidimensional strata. The reliability indices corresponding to the assumed space division were presented graphically in Fig. 3. In the case of maximum 900-piece sample space the reliability index was equal $\beta = 1.295$.

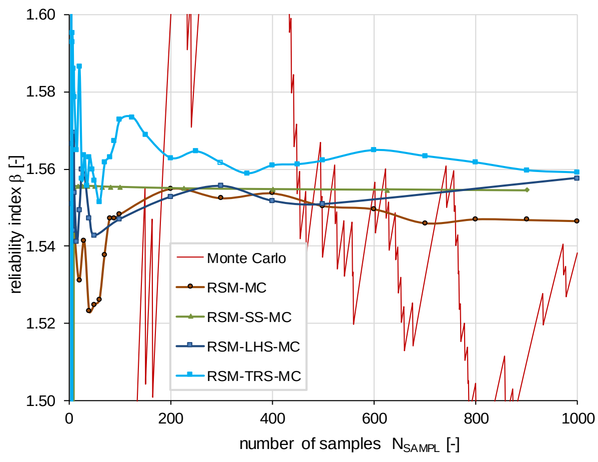

Fig. 3 Computational convergence of reliability index estimated by direct Monte Carlo sampling and response surfaces approximated by points - the results of four various sampling techniques

While Monte Carlo sampling is enhanced by Latin Hypercube technique (MC-LHS) the variable space is split also into identical equidimensional strata (2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2, 15^2, 20^2, 25^2 and 30^2). The reliability index corresponding to the higher strata density in the space of 1000 variables is $\beta = 1.300$. The result convergence related to the number of samples is illustrated in Fig. 3.

6. THE RESPONSE SURFACE METHOD

While the curvature of the real structural response surface is high, the model incorporates the second-order model with interactive terms, in the two-variable case these terms are as follows

$$\hat{g}(\boldsymbol{k}) = B_0 + B_1k_1 + B_2k_2 + B_{11}k_1^2 + B_{22}k_2^2 + B_{12}k_1k_2 \tag{5}$$

The response surface was approximated on the basis of computational points - the results of Monte Carlo sampling (direct variant and the enhanced MC-SS and MC-LHS variants). The response surface was obtained by the dedicated software RSM-Win, whose algorithm was presented in [Winkelmann, 2013].

In the direct Monte Carlo sampling case the response surface was estimated in 19 variants - incorporating a number of 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 300, 400, 500, 600, 700, 800, 900 and 1000 computational points. Coefficients of the polynomial (4) and reliability indices computed in each case were presented in Fig. 3. In the case of 1000 realizations the index $\beta = 1.547$.

Similar computations were conducted due to the points of Monte Carlo sampling enhanced by the Latin Hypercube technique. In this case for 900 samples $\beta = 1.555$. The results corresponding to this sampling variant were presented in Fig. 3.

The results presented in Fig. 3 show that the analysed Monte Carlo variant using 1000 realizations did not lead to the stabilized result. Among the methods incorporating RSM the stratified sampling predicts reliable results even with the use of several samples. All other RSM variants do not make the results stable at the sample number increase. It is worth pointing out that the variations concern the second or third decimal fractional position, thus these various approaches may be considered recursive and stable from an engineering viewpoint.

7. TARGET RANDOM SAMPLING (TRS)

The surface of system mechanical response is determined on the basis of TRS sampling points. In order to do so a dedicated MATLAB procedure was created for sample acquisition in the limit state vicinity $g(k_1, k_2) = 0$. Applying the TRS procedure the space of variables was sectioned into strata (layers), to sample a single point out of each layer. Figure 4 presents an example of computational point generation in the case of 1000 layer division.

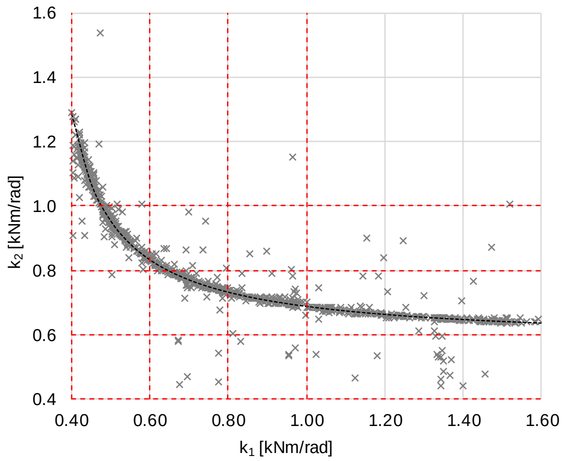

Fig.4. A number of 1000 computational points based on the TRS procedure and the view of assumed planes (red broken lines) intersecting the column response surface

Based on coordinates produced by subsequent divisions the response surface was approximated, next reliability index was obtained. In the case of 1000 realizations $\beta = 1.55913$ (Fig. 3).

Similarly to the previous computations the result vary in a low extent only, at a second or third decimal fractional position. The TRS method proves more reliable, its relevant point generation concerns the limit state function range only (Fig. 4). This analysis is supposed to fit more complex systems of greater result scatter.

Here the system sensitivity analysis is based on the observation of the response surface in selected variability sections of parameters k_1 and k_2. The sensitivity estimation was performed on the basis of 1000 TRS sampling points. The surface equation reads

$$\hat{g}(k_1, k_2) = -0{,}166 + 0{,}162k_1 - 0{,}048k_2 - 0{,}098k_1^2 - \\ -0{,}044k_2^2 + 0{,}248k_1k_2 \tag{6}$$

The layout of intersecting planes is displayed in Fig. 5.

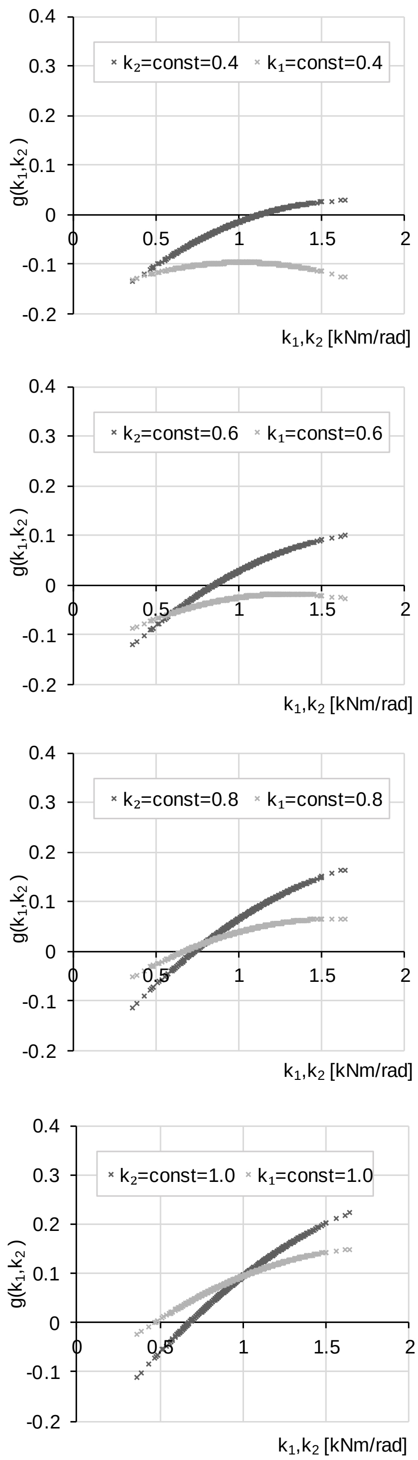

Fig 5 The intersection lines of the system response surface and the planes

The diagrams in Fig. 5 make us conclude that the constant value of a variable k_1 triggers a lower slope of the response surface. Thus it is a parameter to detect higher sensitivity of an input variable. These conclusions are confirmed by former computations of Sobol sensitivity indices.

8. CONCLUSIONS

The Target Sampling Method used in the paper is predicted to be an effective approach. It should be emphasized, that sensitivity analysis presented in the paper is only a preliminary stage to further research in the field of reliability of engineering structures.

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